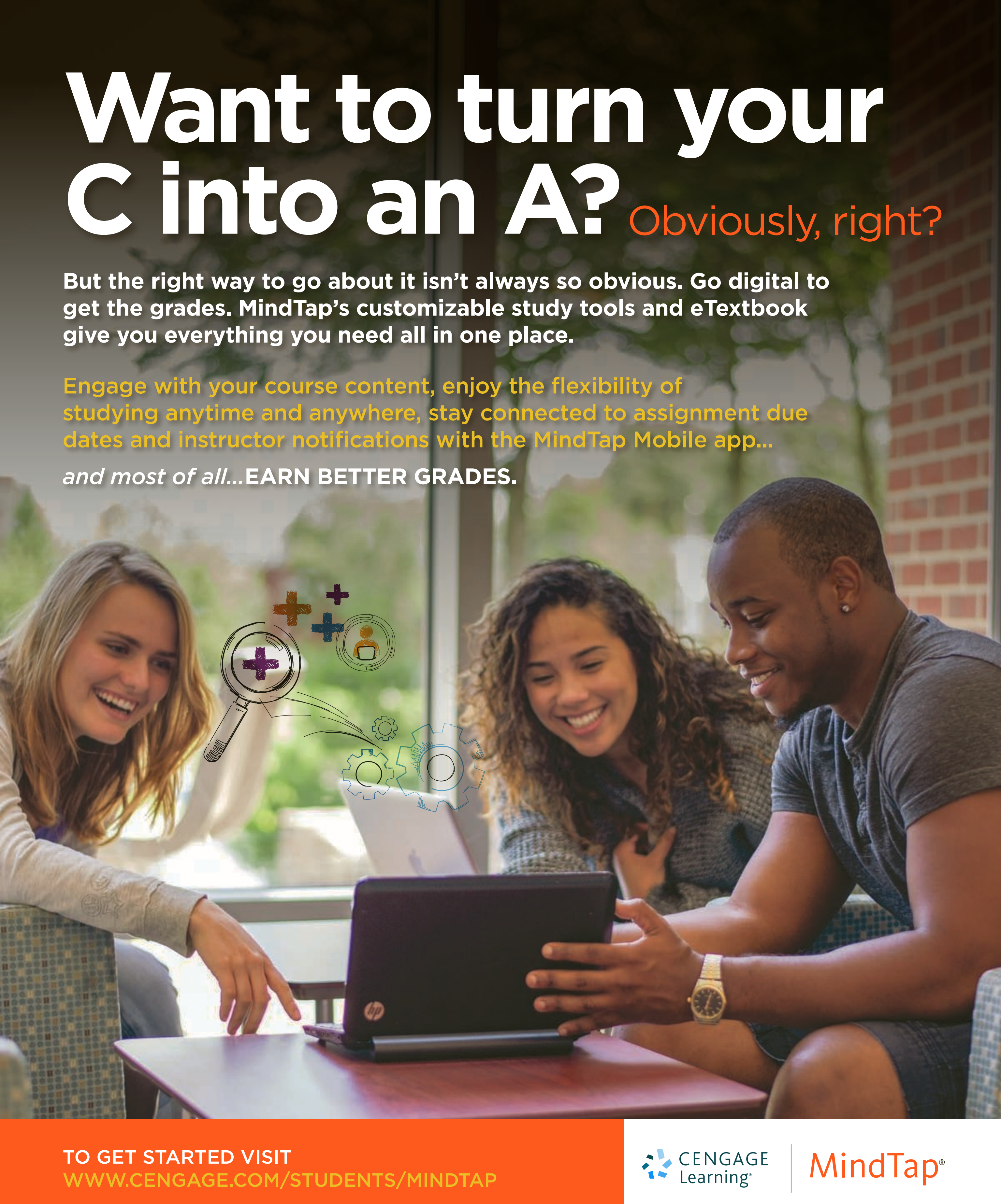
waner + costenoble

finite mathematics 7e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FINITE MATHEMATICS

FINITE MATHEMATICS

SEVENTH EDITION

Stefan Waner
Hofstra University

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Hofstra University

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***Finite Mathematics*, Seventh Edition**
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About the Cover

The cover shows the innermost part of the Large Hadron Collider detector for the ATLAS (a toroidal LHC apparatus) high-energy particle experiment at the CERN Large Hadron Collider in Geneva, Switzerland, before the final insertion of the central pixel detector. The ongoing ATLAS experiment is one of two experiments that led to the discovery of the Higgs boson at CERN in 2012. CERN is the European Organization for Nuclear Research, where physicists and engineers are probing the fundamental structure of the universe.

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Brief Contents

Contents

Preface

Finite Mathematics, Seventh Edition, is intended for a one- or two-term course for students majoring in business, the social sciences, or the liberal arts. Like the earlier editions, the seventh edition of *Finite Mathematics* is designed to address the challenge of generating enthusiasm and mathematical sophistication in an audience that is often underprepared and lacks motivation for traditional mathematics courses. We meet this challenge by focusing on real-life applications that students can relate to, many on topics of current interest; by presenting mathematical concepts intuitively and thoroughly; and by employing a writing style that is informal, engaging, and occasionally even humorous.

The seventh edition goes farther than earlier editions in implementing support for a wide range of instructional paradigms. On the one hand, the abundant pedagogical content available both in print and online, including comprehensive teaching videos and online tutorials, now allows us to be able to offer complete customizable courses for approaches ranging from on-campus and hybrid classes to distance learning classes. In addition, our careful integration of optional support for multiple forms of technology throughout the text makes it adaptable in classes with no technology, classes in which a single form of technology is used exclusively, and classes that incorporate several technologies.

We fully support three forms of technology in this text: TI-83/84 Plus graphing calculators, spreadsheets, and powerful online utilities we have created for the book. In particular, our comprehensive support for spreadsheet technology, both in the text and online, is highly relevant for students who are studying business and economics, in which skill with spreadsheets may be vital to their future careers.

New To This Edition

Content

- **Chapter 0:** We have added an entire new section on logarithms in the Precalculus Review, up through solving for unknowns in the exponent. Students can refer to this section for review when studying techniques involving the use of logarithms in the mathematics of finance (Chapter 2).

- **Chapter 1:** In our revision of this important introductory chapter, we have downplayed the algebra sophistication somewhat so as not to present artificial barriers to the mastery of the important new concepts we discuss.

- **Chapter 2:** The Mathematics of Finance chapter has been significantly revised: In the sections on simple and compound interest, we state and use both the year-based formulas and the compounding period-based versions. In the compound interest section, we now emphasize the latter formulation, as this helps with the segue to annuities, in which the period-based approach is the standard formulation. T-bills and zero coupon bonds are a bit esoteric, so the material on T-bills and further discussion of bonds has been moved to the end of the section to a subsection marked as "Optional." The section on annuities has been substantially reorganized: First, we have standardized the definition of "annuities" and now use more transparent and standard terminology to distinguish accumulation and annuitization (or payout). More important, we have added discussion, examples, and exercises on life insurance and mortgage refinancing, including a formula for calculating principal outstanding. The exercise sets have been radically reorganized and expanded, with numerous real-data based applications that follow the new organization of the section text.

Current Topics in the Applications

- We have added and updated numerous real data exercises and examples based on topics that are either of intense current interest or of general interest to our students, including many on social networks, and the 2009–2016 economic recovery, while retaining those of important historical interest, such as the 2008 economic crisis and resulting stock market panic, and many others.

Exercises

- We have added many new conceptual Communication and Reasoning exercises, including many dealing with common student errors and misconceptions.

Online Visualization and Practice Examples

- We have created a variety of web-based interactive apps available both on **www.wanermath.com** and in the new MindTap course that accompanies this edition. Instructors can use these to demonstrate important concepts such as calculating future values and present values of annuities, graphing inequalities, and solving linear programming problems graphically.
- Many key examples in the text are mirrored by web-based randomizable practice examples, which allow students to test their mastery of the textbook examples and provide instructors with material for interactive presentation and class discussion.

Our Approach to Pedagogy

Real-World Orientation The diversity, breadth, and abundance of examples and exercises included in this edition continue to distinguish our book from others. A large number of these examples and exercises are based on real, referenced data from business, economics, the life sciences, and the social sciences. Our updated examples and exercises in the seventh edition are even more attuned to themes that students can identify with and relate to, from the technology used in their phones and tablets to the social networks in which they participate and many of the corporations they will instantly recognize as important in their lives. Notable events, such as the 1990s dot-com boom, the 2005–2006 real estate bubble, the resulting 2008 economic crisis and stock market panic, and many more, are addressed in examples and exercises throughout the book.

Adapting real data for pedagogical use can be tricky; available data can be numerically complex, intimidating for students, or incomplete. We have modified and streamlined many of the real-world applications, rendering them as tractable as any "made-up" application. At the same time, we have been careful to strike a pedagogically sound balance between applications based on real data and more traditional "generic" applications. Thus, the density and selection of real data-based applications have been tailored to the pedagogical goals and appropriate difficulty level for each section.

Readability We would like students to read this book. We would like students to *enjoy* reading this book. Therefore, we have written the book in a conversational, student-oriented style and have made frequent use of question-and-answer dialogues to encourage the development of the student's mathematical curiosity and intuition. We hope that this text will give the student insight into how a mathematician develops and thinks about mathematical ideas and their applications to real life.

Pedagogical Aids We have included our favorite unique and creative approaches to solving the kinds of problems that normally cause difficulties for students and headaches for instructors. To name just a few, we discuss a rewording technique in Chapters 3 and 5 to show how to translate phrases such as "there are (at least/at most) three times as many X as Y" directly into equations or inequalities, a technique of row reduction in Chapter 3 and tableau manipulation in Chapter 5 based on integer matrices (matrices with fractions are converted to integral matrices in the first step), a zooming-in technique to make solution of traffic flow problems in Chapter 3 almost routine, and decision algorithms in Chapter 6 that make calculations of real-life scenarios involving permutations and combinations almost mechanical.

Rigor Mathematical rigor need not be antithetical to the kind of applied focus and conceptual approach that are hallmarks of this book. We have worked hard to ensure that we are always mathematically honest without being unnecessarily formal. Sometimes we do this through the question-and-answer dialogues and sometimes through the "Before we go on . . ." discussions that follow examples, but always in a manner designed to provoke the interest of the student.

Five Elements of Mathematical Pedagogy to Address Different Learning Styles The "Rule of Four" is a common theme in many texts. Implementing this approach, we discuss many of the central concepts **numerically**, **graphically**, and **algebraically** and clearly delineate these distinctions. The fourth element, **verbal communication** of mathematical concepts, is emphasized through our discussions on translating English sentences into mathematical statements and in our extensive Communication and Reasoning exercises at the end of each section. A fifth element, **interactivity**, is implemented through expanded use of question-and-answer dialogues but is seen most dramatically in the eBook in the MindTap course that accompanies this edition and at **www.wanermath.com** through our new practice and learning modules. These are small interactive apps that help a student visualize new concepts or practice examples similar to those in the text. In addition, the wanermath.com website offers interactive tutorials in the form of games, interactive chapter summaries and chapter review exercises, and online utilities that automate a variety of tasks, from graphing to regression and matrix algebra.

Understand

Examples

Examples are a cornerstone of our approach. Many of the scenarios that we use in application examples and exercises are revisited several times throughout the book. In this way, students will find themselves analyzing the same application from a variety of different perspectives, such as systems of linear equations versus linear programming. Reusing scenarios and important functions provides unifying threads and shows students the complex texture of real-life problems. Complete solutions are provided with every example.

EXAMPLE 1 Savings Accounts

In December 2015, **Radius Bank** was paying 1.10% interest on savings accounts with balances of \$2,500 or more. If the interest is paid as simple interest, find the future value of a \$2,500 deposit after 6 years. What is the total interest paid over the period?[1]

Solution We use the future value formula:

$$\begin{aligned} FV &= PV(1 + rt) \\ &= 2{,}500[1 + (0.011)(6)] = 2{,}500[1.066] = \$2{,}665. \end{aligned}$$

The total interest paid is given by the simple interest formula:

$$\begin{aligned} INT &= PVrt \\ &= (2{,}500)(0.011)(6) = \$165. \end{aligned}$$

Note To find the interest paid, we could also have computed

$$INT = FV - PV = 2{,}665 - 2{,}500 = \$165. \blacksquare$$

Quick Examples

Most definition boxes include quick, straightforward examples that a student can use to solidify each new concept.

Quick Example

1. The simple interest on a \$5,000 investment earning 8% per year for 4 years is

$$\begin{aligned} INT &= PVrt \\ &= (5{,}000)(0.08)(4) = \$1{,}600. \end{aligned}$$

Question-and-Answer Dialogues

We frequently use informal question-and-answer dialogues that anticipate the kinds of questions that may occur to the student and also guide the student through the development of new concepts.

FAQs

Which Formula to Use

Q: *How do I know when to use the formulas based on annual interest, such as $INT = PVrt$, as opposed to those based on general interest periods, such as $INT = PVin$?*

A: You can use either, as convenient. See, for instance, Quick Example 3 and the "Before we go on" discussion after Example 2.

Before We Go On . . .

Most examples are followed by supplementary discussions, which may include a check on the answer, a discussion of the feasibility and significance of a solution, or an in-depth look at what the solution means.

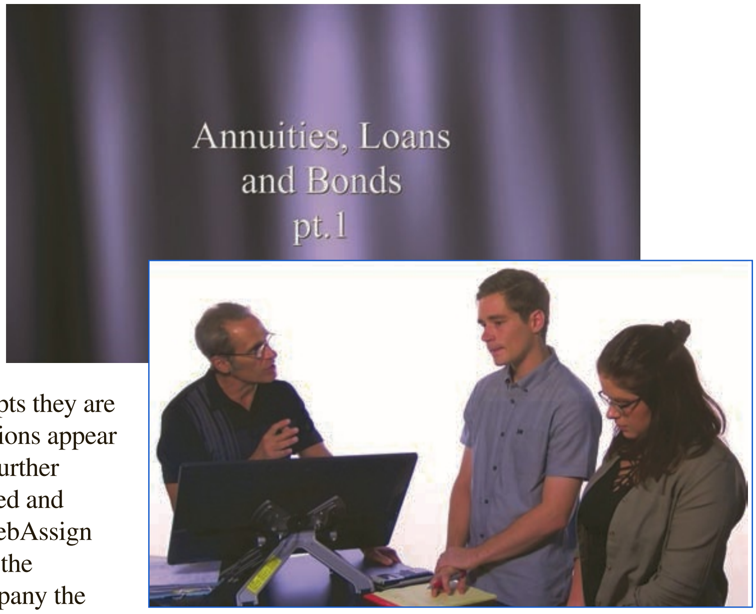
➡ **Before we go on . . .** In Example 1, we could look at the future value as a function of time:

$$FV = 2{,}500(1 + 0.011t) = 2{,}500 + 27.5t.$$

Thus, the future value is growing linearly at a rate of \$27.50 per year ■

Lecture Videos

Developed with Principal Lecturer, Jay Abramson, at Arizona State University, these video clips are flexible in their use as lecture starters in class or as an independent resource for students to review concepts on their own. Blending an introduction to concepts with specific examples, the videos let students quickly see the big picture of key concepts they are learning in class. Selected clips involve students and simulate a classroom-type interaction that creates a sense of the familiar and demystifies key concepts they are learning in their course. Frequently asked questions appear periodically throughout the video segments to further enhance learning. All videos are closed captioned and available in the new MindTap and Enhanced WebAssign courses that accompany the text. The topics for the lecture videos were carefully selected to accompany the subject areas that are most frequently taught and target the concepts that students struggle with most.



Online Visualization and Practice Examples

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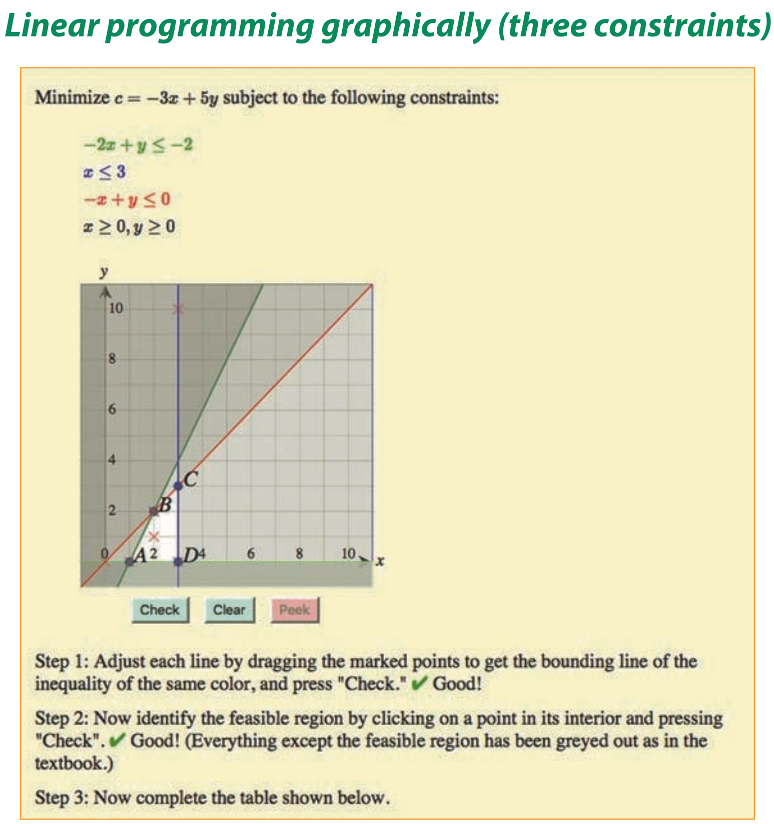
Many key examples in the text are mirrored by web-based randomizable practice examples that allow students to test their mastery of the textbook examples and provide instructors with material for interactive presentation and class discussion.



Practice and Apply

Exercises

Our comprehensive collection of exercises provides a wealth of material that can be used to challenge students at almost every level of preparation and includes everything from straightforward drill exercises to interesting and challenging applications. The exercise sets have been carefully curated and ordered to move from straightforward basic exercises and exercises that are similar to examples in the text to more interesting and advanced ones, marked as "more advanced" for easy reference. There are also several much more difficult exercises, designated as "challenging." We have also included, in virtually every section of every chapter, exercises that are ideal for the use of technology.

2.1 EXERCISES

▼ more advanced ◆ challenging

T indicates exercises that should be solved using technology

In Exercises 1–10, compute the simple interest for the specified length of time and the future value at the end of that time. Round all answers to the nearest cent. [**HINT:** See Quick Examples 1–5.]

1. \$2,000 is invested for 1 year at 6% per year.

2. \$1,000 is invested for 10 years at 4% per year.

3. \$4,000 is invested for 8 months at 0.5% per month.

Application Exercises

Exercises also include interesting applications based on real data to reinforce the applicability of math to real-life situations.

Applications

In Exercises 17–36, compute the specified quantity. Round all answers to the nearest month, the nearest cent, or the nearest 0.001%, as appropriate.

17. ***Simple Loans*** You take out a 6-month, \$5,000 loan at 8% annual simple interest. How much would you owe at the end of the 6 months? [**HINT:** See Example 2.]

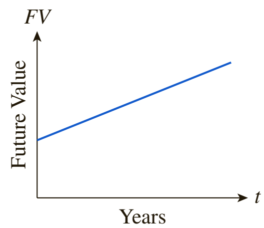
18. ***Simple Loans*** You take out a 15-month, \$10,000 loan at 1% monthly simple interest. How much would you owe at the end of the 15 months? [**HINT:** See Example 2.]

Communication and Reasoning Exercises

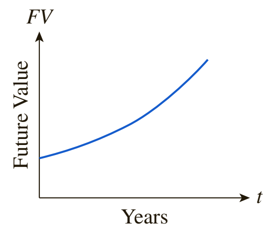
These exercises are designed to help students articulate mathematical concepts, broaden the student's grasp of the mathematical concepts, and develop modeling skills. They include exercises in which the student is asked to provide his or her own examples to illustrate a point or design an application with a given solution. They also include "fill in the blank" type exercises, exercises that invite discussion and debate, and—perhaps most important—exercises in which the student must identify and correct common errors. These exercises often have no single correct answer.

Communication and Reasoning Exercises

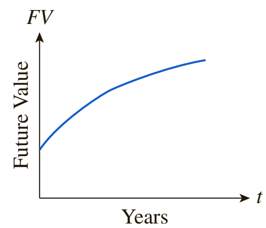
53. One or more of the following three graphs represents the future value of an investment earning simple interest. Which one(s)? Give the reason for your choice(s).



(A)



(B)



(C)

Review

At the end of every chapter is a comprehensive list of the key concepts that were covered in each section.

CHAPTER 2 REVIEW

KEY CONCEPTS

www.WanerMath.com
Go to the Website to find a comprehensive and interactive Web-based summary of Chapter 2.

2.2 Compound Interest
Future value for compound interest:
$FV = PV(1 + i)^n$ [p. 145]
Present value for compound interest:

Payments to accumulate a future value:
$PMT = FV\frac{i}{(1 + i)^n - 1}$ [p. 159]
Annuitization: present value:

Review exercises provide a great way to consolidate and check understanding and prepare for exams.

REVIEW EXERCISES

In Exercises 1–6, find the future value of the investment.

1. \$6,000 for 5 years at 4.75% simple annual interest

2. \$10,000 for 2.5 years at 5.25% simple annual interest

15. The monthly withdrawals possible over 5 years from an account earning 4.75% compounded monthly and starting with \$6,000

Case Studies

Each chapter ends with a section entitled "Case Study," an extended application that uses and illustrates the central ideas of the chapter, focusing on the development of mathematical models appropriate to the topics. These applications are ideal for assignment as projects.

CASE STUDY Adjustable Rate and Subprime Mortgages

Ariel Skelley/Blend Images/Getty Images

The term **subprime mortgage** refers to mortgages given to home buyers with a heightened perceived risk of default, as when, for instance, the price of the home being purchased is higher than the borrower can reasonably afford. Such loans are typically **adjustable rate** loans, meaning that the lending rate varies through the duration of the loan.* Subprime adjustable rate loans typically start at artificially low "teaser rates" that the borrower can afford, but then increase significantly over the life of the mortgage. The U.S. real estate bubble of 2000–2005 led to a frenzy of subprime lending, the rationale being that a borrower having trouble meeting mortgage payments could either sell the property at a profit or refinance the loan, or the lending institution could earn a hefty profit by repossessing the property in the event of foreclosure.

Focus on Technology

Marginal Technology Notes

We give brief marginal technology notes to outline the use of graphing calculator, spreadsheet, and website technology in appropriate examples. When necessary, the reader is referred to more detailed discussion in the end-of-chapter Technology Guides.

End-of-Chapter Technology Guides

We continue to include detailed TI-83/84 Plus and Spreadsheet Guides at the end of each chapter. These Guides are referenced liberally in marginal technology notes at appropriate points in the chapter, so instructors and students can easily use this material or not, as they prefer.

Using Technology

TI-83/84 Plus
$\boxed{\text{APPS}}$ `1:Finance`, then `1:TVM Solver`
N = 120, I% = 5, PV = −5000,
PMT = −100, P/Y = 12, C/Y = 12
With cursor on FV line,
$\boxed{\text{ALPHA}}$ $\boxed{\text{SOLVE}}$
[More details in the Technology Guide.]

Spreadsheet
`=FV(5%/12,10*12,-100,-5000)`
[More details in the Technology Guide.]

Website
www.WanerMath.com
→ Online Utilities
→ Time Value of Money Utility

TI-83/84 Plus Technology Guide

Section 2.2

Example 1 (page 147) In December 2015, Radius Bank was paying 1.10% annual interest on savings accounts with balances of \$2,500 and up. If the interest is compounded quarterly, find the future value of a \$2,500 deposit after 6 years. What is the total interest paid over the time of the investment?

Solution

We could calculate the future value using

(such as the future value of your deposit, which the bank will give back to you) will be a positive number.

Spreadsheet Technology Guide

Section 2.2

Example 1 (page 147) In December 2015, **Radius Bank** was paying 1.10% annual interest on savings accounts with balances of \$2,500 and up. If the interest is compounded quarterly, find the future value of a

`=FV(i,n,PMT,PV)`

i = Interest per period — We use B2/B7 for the interest.

n = Number of periods — We use B3*B7 for the number of

Instructor Resources

MindTap: Through personalized paths of dynamic assignments and applications, MindTap is a digital learning solution and representation of your course that turns cookie cutter into cutting edge, apathy into engagement, and memorizers into higher-level thinkers.

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Personalization: Customize every element of your course—from rearranging the learning path to inserting videos and activities.

Improved Workflow: Save time when planning lessons with all of the trusted, most current content you need in one place in MindTap.

Tracking Students' Progress in Real Time: Promote positive outcomes by tracking students in real time and tailoring your course as needed based on the analytics.

Learn more at **www.cengage.com/mindtap**.

WebAssign: Exclusively from Cengage Learning, Enhanced WebAssign combines the exceptional mathematics content that you know and love with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and eBooks, helping students to develop a deeper conceptual understanding of their subject matter. Quick Prep and Just In Time exercises provide opportunities for students to review prerequisite skills and content, both at the start of the course and at the beginning of each section. Flexible assignment options give instructors the ability to release assignments conditionally on the basis of students' prerequisite assignment scores. Visit us at **www.cengage.com/ewa** to learn more.

Cognero: Cengage Learning Testing Powered by Cognero is a flexible, online system that allows you to author, edit, and manage test bank content; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you choose.

Instructor Companion Site: This collection of book-specific lecture and class tools is available online at **www.cengage.com/login**. Access and download PowerPoint presentations, complete solutions manual, and more.

Student Resources

Student Solutions Manual (ISBN: 978-1-337-28047-1): Go beyond the answers—see what it takes to get there and improve your grade! This manual provides worked-out, step-by-step solutions to the odd-numbered problems in the text. You'll have the information you need to truly understand how the problems are solved.

MindTap: MindTap (assigned by the instructor) is a digital representation of your course that provides you with the tools you need to better manage your limited time, stay organized, and be successful. You can complete assignments whenever and wherever you are ready to learn, with course material specially customized for you by your instructor and streamlined in one proven, easy-to-use interface. With an array of study tools, you'll get a true understanding of course concepts, achieve better grades, and lay the groundwork for your future courses. Learn more at **www.cengage.com/mindtap**.

WebAssign: Enhanced WebAssign (assigned by the instructor) provides you with instant feedback on homework assignments. This online homework system is easy to use and includes helpful links to textbook sections, video examples, and problem-specific tutorials.

CengageBrain: Visit **www.cengagebrain.com** to access additional course materials and companion resources. At the cengagebrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where free companion resources can be found.

The Author Website

The authors' website, accessible through **www.wanermath.com**, has been evolving for close to two decades with growing recognition. Students, raised in an environment in which computers suffuse both work and play, can use their web browsers to engage with the material in an active way. The following features of the authors' website are fully integrated with the text and can be used as a personalized study resource:

- **Interactive Tutorials** Highly interactive tutorials are included on major topics, with guided exercises that parallel the text and a great deal of help and feedback to assist the student.
- **Game Versions of Tutorials** More challenging tutorials with randomized questions that work as games (complete with "health" scores, "health vials," and an assessment of one's performance at the end of the game) are offered alongside the traditional tutorials. These game tutorials, which mirror the traditional "more gentle" tutorials, randomize all the questions and do not give the student the answers but instead offer hints in exchange for "health points," so that just staying alive (not running out of health) can be quite challenging.
- **Learning and Practice Modules** These interactive demos illustrate important concepts and randomizable "practice examples" that mirror many examples and quick examples in the text.
- **Detailed Chapter Summaries** Comprehensive summaries with randomizable interactive elements review all the basic definitions and problem-solving techniques discussed in each chapter. These are a terrific pre-test study tool for students.
- **Downloadable Excel Tutorials** Detailed Excel tutorials are available for almost every section of the book. These interactive tutorials expand on the examples given in the text.
- **Online Utilities** Our collection of easy-to-use online utilities, referenced in the marginal notes of the textbook, allow students to solve many of the technology-based application exercises directly on the web. The utilities include a function grapher and evaluator that also does curve-fitting, regression tools, a time value of money calculator for annuities, a matrix algebra tool that also manipulates matrices with multinomial entries, a linear programming grapher that automatically solves two-dimensional linear programming problems graphically, and a powerful simplex method tool. These utilities require nothing more than a standard web browser.
- **Chapter True-False Quizzes** Randomized quizzes that provide feedback for many incorrect answers based on the key concepts in each chapter assist the student in further mastery of the material.
- **Supplemental Topics** We include complete interactive text and exercise sets for a selection of topics that are not ordinarily included in printed texts but are often requested by instructors.
- **Spanish** A parallel Spanish version of almost the entire website is now deployed, allowing the user to switch languages on specific pages with a single mouse-click. In particular, all of the chapter summaries and most of the tutorials, game tutorials, and utilities are available in Spanish.

Acknowledgments

This project would not have been possible without the contributions and suggestions of numerous colleagues, students, and friends. We are particularly grateful to our colleagues at Hofstra and elsewhere who used and gave us useful feedback on previous editions and suggestions for this one, and to everyone at Cengage for their encouragement and guidance throughout the project. Specifically, we would like to thank Rita Lombard and Morgan Mendoza for their unflagging enthusiasm, Scott Barnett of Henry Ford Community College for his meticulous check of the mathematical accuracy, and Martha Emry and Teresa Trego for whipping the book into shape. Additionally, we would like to thank the creative force of Jay Abramson of Arizona State University for developing the new lecture videos that accompany our text, and Scott Barnett of Henry Ford Community College, Joe Rody of Arizona State University, Nada Al-Hanna of University of Texas at El Paso, and Kaat Higham of Bergen Community College for their thoughtful reviews and input into the scripts.

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FINITE MATHEMATICS

0

PRECALCULUS REVIEW

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Introduction

In this chapter we review some topics from algebra that you need to know to get the most out of this book. This chapter can be used either as a refresher course or as a reference.

There is one crucial fact you must always keep in mind: The letters used in algebraic expressions stand for numbers. All the rules of algebra are just facts about the arithmetic of numbers. If you are not sure whether some algebraic manipulation you are about to do is legitimate, try it first with numbers. If it doesn't work with numbers, it doesn't work.

0.1 Real Numbers

The **real numbers** are the numbers that can be written in decimal notation, including those that require an infinite decimal expansion. The set of real numbers includes all integers, positive, negative, and zero; all fractions; and the irrational numbers, that is, those with decimal expansions that never repeat. Examples of irrational numbers are

$$\sqrt{2} = 1.414213562373 \ldots$$

and

$$\pi = 3.141592653589 \ldots$$

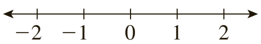

Figure 1

It is very useful to picture the real numbers as points on a line. As shown in Figure 1, larger numbers appear to the right, in the sense that if $a < b$, then the point corresponding to b is to the right of the one corresponding to a.

Intervals

Some subsets of the set of real numbers, called **intervals**, show up quite often, so we have a compact notation for them.

Interval Notation

Here is a list of types of intervals along with examples.

	Interval	Description	Picture	Example
Closed	$[a, b]$	Set of numbers x with $a \le x \le b$	 a b (includes end points)	$[0, 10]$
Open	(a, b)	Set of numbers x with $a < x < b$	 a b (excludes end points)	$(-1, 5)$
Half-Open	$(a, b]$	Set of numbers x with $a < x \le b$	 a b	$(-3, 1]$
	$[a, b)$	Set of numbers x with $a \le x < b$	 a b	$[0, 5)$

Infinite	$[a, +\infty)$	Set of numbers x with $a \le x$		$[10, +\infty)$
	$(a, +\infty)$	Set of numbers x with $a < x$		$(-3, +\infty)$
	$(-\infty, b]$	Set of numbers x with $x \le b$		$(-\infty, -3]$
	$(-\infty, b)$	Set of numbers x with $x < b$		$(-\infty, 10)$
	$(-\infty, +\infty)$	Set of all real numbers		$(-\infty, +\infty)$

Operations

There are five important operations on real numbers: addition, subtraction, multiplication, division, and exponentiation. "Exponentiation" means raising a real number to a power; for instance, $3^2 = 3 \cdot 3 = 9$; $2^3 = 2 \cdot 2 \cdot 2 = 8$.

A note on technology: Most graphing calculators and spreadsheets use an asterisk * for multiplication and a caret ^ for exponentiation. Thus, for instance, 3×5 is entered as `3*5`, $3x$ as `3*x`, and 3^2 as `3^2`.

When we write an expression involving two or more operations, such as

$$2 \cdot 3 + 4$$

or

$$\frac{2 \cdot 3^2 - 5}{4 - (-1)},$$

we need to agree on the order in which to do the operations. Does $2 \cdot 3 + 4$ mean $(2 \cdot 3) + 4 = 10$ or $2 \cdot (3 + 4) = 14$? We all agree to use the following rules for the order in which we do the operations.

Standard Order of Operations

Parentheses and Fraction Bars First, calculate the values of all expressions inside parentheses or brackets, working from the innermost parentheses out, before using them in other operations. In a fraction, calculate the numerator and denominator separately before doing the division.

Quick Examples

1. $6(2 + [3 - 5] - 4) = 6(2 + (-2) - 4) = 6(-4) = -24$
2. $\dfrac{(4 - 2)}{3(-2 + 1)} = \dfrac{2}{3(-1)} = \dfrac{2}{-3} = -\dfrac{2}{3}$
3. $3/(2 + 4) = \dfrac{3}{2 + 4} = \dfrac{3}{6} = \dfrac{1}{2}$
4. $(x + 4x)/(y + 3y) = (5x)/(4y)$

Exponents Next, perform exponentiation.

Quick Examples

5. $2 + 4^2 = 2 + 16 = 18$

6. $(2 + 4)^2 = 6^2 = 36$

Note the difference.

7. $2\left(\dfrac{3}{4-5}\right)^2 = 2\left(\dfrac{3}{-1}\right)^2 = 2(-3)^2 = 2 \times 9 = 18$

8. $2(1 + 1/10)^2 = 2(1.1)^2 = 2 \times 1.21 = 2.42$

Multiplication and Division Next, do all multiplications and divisions, from left to right.

Quick Examples

9. $2(3 - 5)/4 \cdot 2 = 2(-2)/4 \cdot 2$ Parentheses first

$= -4/4 \cdot 2$ Leftmost product

$= -1 \cdot 2 = -2$ Multiplications and divisions, left to right

10. $2(1 + 1/10)^2 \times 2/10 = 2(1.1)^2 \times 2/10$ Parentheses first

$= 2 \times 1.21 \times 2/10$ Exponent

$= 4.84/10 = 0.484$ Multiplications and divisions, left to right

11. $4\dfrac{2(4-2)}{3(-2 \cdot 5)} = 4\dfrac{2(2)}{3(-10)} = 4\dfrac{4}{-30} = \dfrac{16}{-30} = -\dfrac{8}{15}$

Addition and Subtraction Last, do all additions and subtractions, from left to right.

Quick Examples

12. $2(3 - 5)^2 + 6 - 1 = 2(-2)^2 + 6 - 1 = 2(4) + 6 - 1$

$= 8 + 6 - 1 = 13$

13. $\left(\dfrac{1}{2}\right)^2 - (-1)^2 + 4 = \dfrac{1}{4} - 1 + 4 = -\dfrac{3}{4} + 4 = \dfrac{13}{4}$

14. $3/2 + 4 = 1.5 + 4 = 5.5$

15. $3/(2 + 4) = 3/6 = 1/2 = 0.5$

Note the difference.

16. $4/2^2 + (4/2)^2 = 4/2^2 + 2^2 = 4/4 + 4 = 1 + 4 = 5$

17. `-2^4 = (-1)2^4 = -16` A negative sign before an expression means multiplication by -1.[1]

[1] Spreadsheets and some programming languages interpret `-2^4` (wrongly!) as `(-2)^4=16`. So when working with spreadsheets, write `-2^4` as `(-1)*2^4` to avoid this issue.

T indicates material discussing the use of technologies such as graphing calculators, spreadsheets, and web utilities.

T Entering Formulas

Any good calculator or spreadsheet will respect the standard order of operations. However, we must be careful with division and exponentiation and use parentheses as necessary. The following table gives some examples of simple mathematical expressions and their equivalents in the functional format used in most graphing calculators, spreadsheets, and computer programs.

Mathematical Expression	Formula	Comments
$\dfrac{2}{3-x}$	`2/(3-x)`	Note the use of parentheses instead of the fraction bar. If we omit the parentheses, we get the expression shown next.
$\dfrac{2}{3}-x$	`2/3-x`	The calculator follows the usual order of operations.
$\dfrac{2}{3\times 5}$	`2/(3*5)`	Putting the denominator in parentheses ensures that the multiplication is carried out first. The asterisk is usually used for multiplication in graphing calculators and computers.
$\dfrac{2}{x}\times 5$	`(2/x)*5`	Putting the fraction in parentheses ensures that it is calculated first. Some calculators will interpret `2/3*5` as $\dfrac{2}{3\times 5}$ but `2/3(5)` as $\dfrac{2}{3}\times 5$.
$\dfrac{2-3}{4+5}$	`(2-3)/(4+5)`	Note once again the use of parentheses in place of the fraction bar.
2^3	`2^3`	The caret ^ is commonly used to denote exponentiation.
2^{3-x}	`2^(3-x)`	Be careful to use parentheses to tell the calculator where the exponent ends. Enclose the *entire exponent* in parentheses.
2^3-x	`2^3-x`	Without parentheses, the calculator will follow the usual order of operations: exponentiation and then subtraction.
3×2^{-4}	`3*2^(-4)`	On some calculators, the negation key is separate from the minus key.
$2^{-4\times 3}\times 5$	`2^(-4*3)*5`	Note once again how parentheses enclose the entire exponent.
$100\left(1+\dfrac{0.05}{12}\right)^{60}$	`100*(1+0.05/12)^60`	This is a typical calculation for compound interest.
$PV\left(1+\dfrac{r}{m}\right)^{mt}$	`PV*(1+r/m)^(m*t)`	This is the compound interest formula. *PV* is understood to be a single number (present value) and not the product of *P* and *V* (or else we would have used `P*V`).

$\dfrac{2^{3-2}\times 5}{y-x}$	`2^(3-2)*5/(y-x)`
	or
	`(2^(3-2)*5)/(y -x)`
		Notice again the use of parentheses to hold the denominator together. We could also have enclosed the numerator in parentheses, although this is optional. (Why?)

$\dfrac{2^y+1}{2-4^{3x}}$	`(2^y+1)/(2-4^(3*x))`	Here, it is necessary to enclose both the numerator and the denominator in parentheses.
$2^y+\dfrac{1}{2}-4^{3x}$	`2^y+1/2-4^(3*x)`	This is the effect of leaving out the parentheses around the numerator and denominator in the previous expression.

Accuracy and Rounding

When we use a calculator or computer, the results of our calculations are often given to far more decimal places than are useful. For example, suppose we are told that a square has an area of 2.0 square feet and we are asked how long its sides are. Each side is the square root of the area, which the calculator tells us is

$$\sqrt{2} \approx 1.414213562.$$

However, the measurement of 2.0 square feet is probably accurate to only two digits, so our estimate of the lengths of the sides can be no more accurate than that. Therefore, we round the answer to two digits:

$$\text{Length of one side} \approx 1.4 \text{ feet.}$$

The digits that follow 1.4 are meaningless. The following guide makes these ideas more precise.

Significant Digits, Decimal Places, and Rounding

The number of **significant digits** in a decimal representation of a number is the number of digits that are not leading zeros after the decimal point (as in .0005) or trailing zeros before the decimal point (as in 5,400,000). We say that a value is **accurate to *n* significant digits** if only the first n significant digits are meaningful.

When to Round

After doing a computation in which all the quantities are accurate to no more than n significant digits, round the final result to n significant digits.

Quick Examples

18. 0.00067 has two significant digits. — The 000 before 67 are leading zeros.

19. 0.000670 has three significant digits. — The 0 after 67 is significant.

20. 5,400,000 has two or more significant digits. — We can't say how many of the zeros are trailing.[2]

21. 5,400,001 has seven significant digits. — The string of zeros is not trailing.

22. Rounding 63,918 to three significant digits gives 63,900.

23. Rounding 63,958 to three significant digits gives 64,000.

24. $\pi = 3.141592653\ldots \quad \frac{22}{7} = 3.142857142\ldots$ Therefore, $\frac{22}{7}$ is an approximation of π that is accurate to only three significant digits: 3.14.

25. $4.02(1 + 0.02)^{1.4} \approx 4.13$ — We rounded to three significant digits.

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[2] If we obtained 5,400,000 by rounding 5,401,011, then it has three significant digits because the zero after the 4 is significant. On the other hand, if we obtained it by rounding 5,411,234, then it has only two significant digits. The use of scientific notation avoids this ambiguity: 5.40×10^6 (or `5.40E6` on a calculator or computer) is accurate to three digits, and 5.4×10^6 is accurate to two digits.

One more point, though: If, in a long calculation, you round the intermediate results, your final answer may be even less accurate than you think. As a general rule,

When calculating, don't round intermediate results. Rather, use the most accurate results obtainable, or have your calculator or computer store them for you.

When you are done with the calculation, *then* round your answer to the appropriate number of digits of accuracy.

0.1 EXERCISES

Calculate each expression in Exercises 1–24, giving the answer as a whole number or a fraction in lowest terms.

1. $2(4+(-1))(2\cdot -4)$

2. $3+([4-2]\cdot 9)$

3. `20/(3*4)-1`

4. `2-(3*4)/10`

5. $\dfrac{3+([3+(-5)])}{3-2\times 2}$

6. $\dfrac{12-(1-4)}{2(5-1)\cdot 2-1}$

7. `(2-5*(-1))/1-2*(-1)`

8. `2-5*(-1)/(1-2*(-1))`

9. $2\cdot(-1)^2/2$

10. $2+4\cdot 3^2$

11. $2\cdot 4^2+1$

12. $1-3\cdot(-2)^2\times 2$

13. `3^2+2^2+1`

14. `2^(2^2-2)`

15. $\dfrac{3-2(-3)^2}{-6(4-1)^2}$

16. $\dfrac{1-2(1-4)^2}{2(5-1)^2\cdot 2}$

17. `10*(1+1/10)^3`

18. `121/(1+1/10)^2`

19. $3\left(\dfrac{-2\cdot 3^2}{-(4-1)^2}\right)$

20. $-\left(\dfrac{8(1-4)^2}{-9(5-1)^2}\right)$

21. $3\left(1-\left(-\dfrac{1}{2}\right)^2\right)^2+1$

22. $3\left(\dfrac{1}{9}-\left(\dfrac{2}{3}\right)^2\right)^2+1$

23. `(1/2)^2-1/2^2`

24. `2/(1^2)-(2/1)^2`

Convert each expression in Exercises 25–50 into its technology formula equivalent as in the table in the text.

25. $3\times(2-5)$

26. $4+\dfrac{5}{9}$

27. $\dfrac{3}{2-5}$

28. $\dfrac{4-1}{3}$

29. $\dfrac{3-1}{8+6}$

30. $3+\dfrac{3}{2-9}$

31. $3-\dfrac{4+7}{8}$

32. $\dfrac{4\times 2}{\left(\frac{2}{3}\right)}$

33. $\dfrac{2}{3+x}-xy^2$

34. $3+\dfrac{3+x}{xy}$

35. $3.1x^3-4x^{-2}-\dfrac{60}{x^2-1}$

36. $2.1x^{-3}-x^{-1}+\dfrac{x^2-3}{2}$

37. $\dfrac{\left(\frac{2}{3}\right)}{5}$

38. $\dfrac{2}{\left(\frac{3}{5}\right)}$

39. $3^{4-5}\times 6$

40. $\dfrac{2}{3+5^{7-9}}$

41. $3\left(1+\dfrac{4}{100}\right)^{-3}$

42. $3\left(\dfrac{1+4}{100}\right)^{-3}$

43. $3^{2x-1}+4^x-1$

44. $2^{x^2}-(2^{2x})^2$

45. 2^{2x^2-x+1}

46. $2^{2x^2-x}+1$

47. $\dfrac{4e^{-2x}}{2-3e^{-2x}}$

48. $\dfrac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

49. $3\left(1-\left(-\dfrac{1}{2}\right)^2\right)^2+1$

50. $3\left(\dfrac{1}{9}-\left(\dfrac{2}{3}\right)^2\right)^2+1$

0.2 Exponents and Radicals

In Section 0.1 we discussed exponentiation, or "raising to a power"; for example, $2^3=2\cdot 2\cdot 2$. In this section we discuss the algebra of exponentials more fully. First, we look at *integer* exponents: cases in which the powers are positive or negative whole numbers.

Integer Exponents

Positive Integer Exponents

If a is any real number and n is any positive integer, then by a^n we mean the quantity $a \cdot a \cdot \ldots \cdot a$ (n times); thus, $a^1 = a$, $a^2 = a \cdot a$, $a^5 = a \cdot a \cdot a \cdot a \cdot a$. In the expression a^n the number n is called the **exponent**, and the number a is called the **base**.

Quick Examples

$3^2 = 9$ $\qquad$ $2^3 = 8$

$0^{34} = 0$ $\qquad$ $(-1)^5 = -1$

$10^3 = 1{,}000$ $\qquad$ $10^5 = 100{,}000$

Negative Integer Exponents

If a is any real number *other than zero* and n is any positive integer, then we define

$$a^{-n} = \frac{1}{a^n} = \frac{1}{a \cdot a \cdot \ldots \cdot a} \quad (n \text{ times}).$$

Quick Examples

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $\qquad$ $1^{-27} = \frac{1}{1^{27}} = 1$

$x^{-1} = \frac{1}{x^1} = \frac{1}{x}$ $\qquad$ $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

$y^7 y^{-2} = y^7 \frac{1}{y^2} = y^5$ $\qquad$ 0^{-2} is not defined

Zero Exponent

If a is any real number other than zero, then we define

$$a^0 = 1.$$

Quick Examples

$3^0 = 1$ $\qquad$ $1{,}000{,}000^0 = 1$

0^0 is not defined

When combining exponential expressions, we use the following identities.

Exponent Identity

1. $a^m a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}$ if $a \neq 0$

3. $(a^n)^m = a^{nm}$

4. $(ab)^n = a^n b^n$

5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ if $b \neq 0$

Quick Examples

1. $2^3 2^2 = 2^{3+2} = 2^5 = 32$

$x^3 x^{-4} = x^{3-4} = x^{-1} = \dfrac{1}{x}$

$\dfrac{x^3}{x^{-2}} = x^3 \dfrac{1}{x^{-2}} = x^3 x^2 = x^5$

2. $\dfrac{4^3}{4^2} = 4^{3-2} = 4^1 = 4$

$\dfrac{x^3}{x^{-2}} = x^{3-(-2)} = x^5$

$\dfrac{3^2}{3^4} = 3^{2-4} = 3^{-2} = \dfrac{1}{9}$

3. $(3^2)^2 = 3^4 = 81$

$(2^x)^2 = 2^{2x}$

4. $(4 \cdot 2)^2 = 4^2 2^2 = 64$

$(-2y)^4 = (-2)^4 y^4 = 16y^4$

5. $\left(\dfrac{4}{3}\right)^2 = \dfrac{4^2}{3^2} = \dfrac{16}{9}$

$\left(\dfrac{x}{-y}\right)^3 = \dfrac{x^3}{(-y)^3} = -\dfrac{x^3}{y^3}$

Caution

- In the first two identities, the bases of the expressions must be the same. For example, the first identity gives $3^2 3^4 = 3^6$ but does *not* apply to $3^2 4^2$.
- People sometimes invent their own identities, such as $a^m + a^n = a^{m+n}$, which is wrong! (Try it with $a = m = n = 1$.) If you wind up with something like $2^3 + 2^4$, you are stuck with it; there are no identities around to simplify it further. (You can factor out 2^3, but whether or not that is a simplification depends on what you are going to do with the expression next.)

EXAMPLE 1 Combining the Identities

$$\begin{aligned}
\frac{(x^2)^3}{x^3} &= \frac{x^6}{x^3} && \text{By identity (3)}\\
&= x^{6-3} && \text{By identity (2)}\\
&= x^3
\end{aligned}$$

$$\begin{aligned}
\frac{(x^4 y)^3}{y} &= \frac{(x^4)^3 y^3}{y} && \text{By identity (4)}\\
&= \frac{x^{12} y^3}{y} && \text{By identity (3)}\\
&= x^{12} y^{3-1} && \text{By identity (2)}\\
&= x^{12} y^2
\end{aligned}$$